ANOMALOUS SPINORS AND BOSONS

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A discussion is given of spinors and bosons which behave unusually under inversions and which in particular illustrate the anomalous commutation relations given by Gel'fand and Tsetlin. Possibilities of setting up such wave functions for various particles are indicated.

THE discovery of the isotopic-spin and strangeness properties of particles and the discovery of parity nonconservation in the weak interactions show the need for a more careful study of the properties of spinors and bosons under inversions. The usual representations of the Lorentz group are inadequate for the description of the whole extent of new properties of particles. In this connection the theory of isotopic space (three-dimensional or four-dimensional) has been developed.[1] Also it is found that the application of previously neglected possibilities of projective representations in the theory of spinors and "integrons" (particles of integer spin) even offers hopes of arriving at a description of isotopic spin and strangeness within the framework of ordinary space.[2] These questions acquire importance in applications of the method of fusion[3] and of compound-particle models,[4] and even in connection with the program of the nonlinear theory of matter,[5] in which unusual types of spinors have to be taken into account as possible elements of the field. Our problem is the analysis of the new types of spinors, and also the indication of unusual bilinear combinations and the examination of spin and statistics in this context.

We shall list the notations to be used:

$$ab = a_\mu b_\mu = a_i b_i - a_0 b_0, \quad (i = 1, 2, 3),$$

$$\gamma_i = -\gamma_i^+ = \begin{pmatrix} 0 & i\sigma_i \\ i\sigma_i & 0 \end{pmatrix}, \quad \gamma_0 = \gamma_0^+ = \begin{pmatrix} 0 & iE_2 \\ -iE_2 & 0 \end{pmatrix},$$

$$\gamma_5 = \begin{pmatrix} -iE_2 & 0 \\ 0 & iE_2 \end{pmatrix}. \tag{1}$$

The charge-conjugate spinor is best defined by the "anti-involution" relation

$$\psi^{C'} = \gamma_2\psi^*, \quad \psi = -\gamma_2\psi^{C'*} \tag{2}$$

It is also possible to use the more common relation[6]

$$\psi^C = i\gamma_5\psi^{C'} = i\gamma_5\gamma_2\psi^* = C\psi^*, \quad \psi = i\gamma_5\gamma_2\psi^{C*}. \tag{2'}$$

In both cases the charge-conjugate spinors transform apart from sign like the original spinors. The definition (2) possesses an advantage in view of the theorem: "Under the substitution $\psi \to \psi^{C'}$ every bilinear combination $\psi^+\gamma\psi$, $\tilde{\psi}\gamma\psi$, etc. (γ is a 4×4 matrix) goes over into the complex-conjugate quantity" (in the case of (2′) we get $\pm$ the complex conjugate for $\psi \to \psi^C$). Thus the change from ψ to $\psi^{C'}$ is a generalization of complex conjugation.

Let us denote the product of the three space reflections by P and the geometric time reversal by T^0. Lagrangians must be invariant with respect to P and the Schwinger inversion $T' = T^0 \times (\sim)$, where the tilde denotes transposition of operators in Hilbert space.[6] Invariance under T' is in the last analysis equivalent to the condition of invariance under T^0 of the time-ordered T-products of operators in the Heisenberg representation. In virtue of the Hermitian property we have for the Lagrangian

$$T' = T^0 \times (\sim) = T^0 \times (*). \tag{3}$$

In its turn the operation of complex conjugation (*) reduces to two changes: first, replacement of operators by the charge-(complex)-conjugate operators in the sense of Eq. (2), and second, replacement of i by −i.

Let us list the various possibilities for the behavior of spinors under the geometrical reflections of the coordinates and the time (under rotations preserving the sense of the time all spinors have the same behavior), giving besides the previously known basic types I and II[8,9] the spinors of types III and IV that we have pointed out:[10]

"normal" spinors (type I)

$$\psi^{11} \to \pm\gamma_5 a_\mu \gamma_\mu \psi^{11}, \quad a^2 = \pm 1,$$

"pseudospinors" (type II)

$$\psi^{22} \rightarrow \pm a_\mu \gamma_\mu \psi^{22}, \quad a^2 = \pm 1,$$

"mixed" spinors (type III)

$$\psi^{12} \rightarrow \begin{cases} \pm \gamma_5 a_\mu \gamma_\mu \psi^{12}, & a^2 = +1 \\ \pm a_\mu \gamma_\mu \psi^{12}, & a^2 = -1, \end{cases} \tag{4}$$

and "mixed" spinors (type IV)

$$\psi^{21} \rightarrow \begin{cases} \pm a_\mu \gamma_\mu \psi^{21} & a^2 = +1 \\ \pm \gamma_5 a_\mu \gamma_\mu \psi^{21} & a^2 = -1 \end{cases}$$

(a_μ is the unit vector normal to the hypersurface with respect to which the reflection occurs; the first of the upper indices on the refers to space reflection, the second to time reflection).

We must further allow for the possibility of introducing additional factors $\pm i$ as well as ± 1 for space and time reflections (individually and collectively)* for all four fundamental types of spinors, in the same way as the factors ± 1, $\pm i$ were introduced earlier for space inversion of spinors of the type ψ^{11}.† Generalizing the Yang-Tiomno notation, we can introduce, for example, a spinor ψ^{1A2D} which transforms under space reflections by the matrix

$$+\gamma_5 a_\mu \gamma_\mu, \quad a_\mu^2 = +1, \tag{5.1}$$

and under time reflection by the matrix

$$-i a_\mu \gamma_\mu, \quad a_\mu^2 = -1 \text{ and so on} \tag{5.2}$$

Thus one gets 64 distinct types of spinors. As can easily be verified, for all spinors of types A, B, (4) the square of a space reflection is $P^2 = -1$ (or a rotation through 2π), and the square of a time reflection is $T^2 = 1$; for spinors C, D, $P^2 = 1$, $T^2 = -1$; for spinors AC, AD, BC, BD, CA, etc. (for meanings of notations A, B, C, D see the paper of Yang and Tiomno[11]), $P^2 = T^2 = \pm 1$. Self-adjoint (Majorana) spinors that do not vanish identically can only be of types CD, DC, CC, DD, since in this case, following the inversion invariance, we can set

$$\psi = \Omega \psi^c = \Omega i \gamma_5 \gamma_2 \psi^*, \quad |\Omega|^2 = 1. \tag{6}$$

For the spinors of types I and II with homogeneous behavior (of any of the classes A, B, C, D)

$$PT = -TP,$$

but for the "mixed" spinors ψ^{12} and ψ^{21} (classes A, B, C, D)

$$PT = TP \tag{6'}$$

(one can choose P = T).

Thus we can say that our "mixed" spinors directly realize what can reasonably be called the "anomalous" (cf. reference 7) four-dimensional spinor representation; for them the operations P and T commute, whereas for the "normal" spinors ψ^{11} and ψ^{22} of types I and II (all classes A, B, C, D) these operators anticommute.*

"Normal" spinors of the class ψ^{1A1A} are ordinarily assigned to the electron-positron field.

Under space reflections the bilinear combinations

$$\psi^+ \gamma_0 \gamma \psi, \tag{7}$$

$$\psi \gamma_2 \gamma_0 \gamma \psi, \tag{7'}$$

formed from spinors ψ^{11} (classes A, B) have, as is well known, the behavior of a scalar, a vector, an antisymmetric tensor, a pseudovector, and a pseudoscalar. Under time reflections ($a^2 = -1$) the covariants (7) and (7′) (classes A, B) get an additional factor -1, i.e., the scalar behaves like a pseudoscalar, and so on. Combinations of the type (7) formed from spinors ψ^{11} of classes BC, AD, AC, and so on, transform in an analogous way under inversions. Meanwhile the combinations (7′) formed from spinors of these latter classes get an additional factor -1 under an inversion.

Thus the Lagrangian of a linear theory formed in the usual way from spinors of type I will be a scalar with respect to the operation P and a pseudovector with respect to the operation T^0. Therefore in order for the Lagrangian to be unchanged under the operation T′ it must get multiplied by -1 by the operation of transposition (~), or, what is the same thing, the operation of complex conjugation (*). As the result we get, as usual, the Lagrangian† (we have written out as an illustration the case of interaction with the electromagnetic field; the indices 11, AB, CD, etc. are omitted):

*Here it must be remembered that the cases $\pm i$, ± 1 refer to representations of the complete Lorentz group, as supplemented with the reflections in different ways.

†In a number of cases the transformation matrices of the various spinors are equivalent under unitary transformation, and we can speak only of a relative difference. For example, ψ^{11} (A) and ψ^{22} (A), etc., or ψ^{12} (A) and ψ^{21} (A), etc., are equivalent in the usual sense, with the unitary transformation matrix $u = (1+\gamma_5)/2^{1/2}$. When, however, we take into account the antilinear relations associated with the antiparticle conjugation, the equivalence is destroyed. In fact, $CA^* = -AC$ for $A = a_\mu \gamma_\mu$, but $C'A'^* = +A'C'$ for $A' = a_\mu \gamma_\mu \gamma_5 = 2^{-1/2}(1+\gamma_5)A \cdot 2^{-1/2}(1-\gamma_5)$, $C' = 2^{-1/2}(1+\gamma_5)C \cdot 2^{-1/2}(1-\gamma_5)$. Examination of the auxiliary pseudospinors defined by Cartan in a space of an arbitrary number of dimensions is especially useful for the introduction of the "mixed" spinors.

*We note that the "mixed" spinors ψ^{12} and ψ^{21} the well known formal difficulties[8] with the construction of a spinor theory in a Riemannian space are absent.

†In Eq. (8) the expressions are equal part from a divergence. We set $\hbar = c = 1$.

$$L = {}^1/_2\Big[(\gamma_0\gamma_\alpha)_{\alpha\beta}\Big(\psi^*_\alpha i\frac{\partial\psi_\beta}{\partial x^\mu} - i\frac{\partial\psi_\beta}{\partial x^\mu}\psi^*_\alpha\Big) + m(\gamma_0)_{\alpha\beta}(\psi^*_\alpha\psi_\beta - \psi_\beta\psi^*_\alpha)$$
$$+ eA_\mu(\gamma_0\gamma_\mu)_{\alpha\beta}(\psi^*_\alpha\psi_\beta - \psi_\beta\psi^*_\alpha)\Big]$$
$$= {}^1/_2[\psi^+\gamma_0\gamma_\mu i\partial\psi/\partial x^\mu + \psi^{C'+}\gamma_0\gamma_\mu i\partial\psi^{C'}/\partial x^\mu$$
$$+ eA_\mu(\psi^+\gamma_0\gamma_\mu\psi - \psi^{C'+}\gamma_0\gamma_\mu\psi^{C'}) + m(\psi^+\gamma_0\psi - \psi^{C'+}\gamma_0\psi^{C'})]$$
$$= {}^1/_2[(\psi^+\gamma_0\gamma_\mu i\partial\psi/\partial x^\mu + \psi^{C+}\gamma_0\gamma_\mu i\partial\psi^C/\partial x^\mu)$$
$$+ eA_\mu(\psi^+\gamma_0\gamma_\mu\psi - \psi^{C+}\gamma_0\gamma_\mu\psi^C) + m(\psi^+\gamma_0\psi + \psi^{C+}\gamma_0\psi^C)]. \quad (8)$$

As can be seen from the first form given in Eq. (8), L changes sign under the operation of transposition, and consequently if it were the commutator, and not the anticommutor, of ψ and ψ^* that is a c-number the entire Lagrangian would reduce to a c-number. The arguments given are equivalent to a proof of the theorem of the connection between spin and statistics.[6]

For type I spinors ψ^{1A1A}, etc., the Dirac equation has the usual form

$$-i\gamma_\mu\partial\psi/\partial x^\mu + eA_\mu\gamma_\mu\psi + m\psi = 0. \quad (9)$$

In the case of the type II* pseudospinors ψ^{2A2A}, etc., the space vector and pseudovector (7) will have the same structures as for ψ^{11}, whereas the space scalar and pseudoscalar (7) are interchanged. Accordingly a γ_5 enters the mass term of the expression (8), and an equation of the Dirac type is written in invariant form as follows:[9,10]

$$-i\gamma_\mu\partial\psi/\partial x^\mu + eA_\mu\gamma_\mu\psi + \gamma_5 m\psi = 0, \quad (10)$$

and here the Hermitian character of the Lagrangian is preserved (despite reference 9), and also the connection between energy-momentum and mass,

$$\varepsilon^2 - \mathbf{p}^2 - m^2 = 0. \quad (11)$$

For type II spinors, which are normal, the proof of their satisfying Fermi statistics goes through just as for type I spinors, since, as follows from Eq. (4), for them L is a scalar with respect to P and a pseudoscalar with respect to T^0.

Let us introduce the operation of charge (antiparticle) conjugation

$$C: \psi\to\psi^{C'}, \quad e\to -e, \quad i\to i, \quad m\to -m,$$
$$A_\nu\to A_{\nu'}, \quad \varphi\to\varphi^* \quad (12)$$

or in more usual form

$$C: \quad \psi\to\psi^C, \quad e\to -l, \quad i\to i,$$
$$m\to m, \quad \varphi\to\varphi^* \quad (13)$$

In its action on the Lagrangian C differs from complex conjugation, and consequently also from transposition, by a factor -1; symbollically we can write

$$(\sim) = (*) = C\times(-1). \quad (14)$$

For normal spinors the operations ($\sim$) and ($*$) change the sign of L, so as to compensate for the change of sign produced by T^0. Thus linear-theory Lagrangians for spinors of types I and II are automatically invariant with respect to C (in any case if we do not include the four-fermion interactions). This fact forms a part, applicable to normal particles, of the PCT (or PT′) theorem of Lüders, where the Wigner inversion T = CT′ is the time inversion T^0 with multiplication by -1 (for Hermitian quantities).

Let us pass to the consideration of the mixed spinors, for which the space scalar and pseudoscalar will also behave like a scalar and pseudoscalar, respectively, under the operation T^0. At the same time, just as in the case of the normal spinors, the space vector and pseudovector will behave under T^0 like a pseudovector and vector, respectively. From this follows the impossibility of constructing, without additional assumptions, an inversion-invariant Lagrangian for the anomalous spinors with a mass term that does not reduce to a c-number, and also a violation of the Lüders theorem. In this connection there are three possibilities. First, we could renounce invariance with respect to P, while preserving invariance with respect to T′. Then, for example for ψ^{12}, we can write the Lagrangian in the form

$$L = {}^1/_2\Big[(\gamma_0\gamma_\mu)_{\alpha\beta}\Big(\psi^*_\alpha i\frac{\partial\psi_\beta}{\partial x^\mu} - i\frac{\partial\psi_\beta}{\partial x^\mu}\psi^*_\alpha\Big) + m(\gamma_0\gamma_5)_{\alpha\beta}(\psi^*_\alpha\psi_\beta - \psi_\beta\psi^*_\alpha)$$
$$+ eA_\mu(\gamma_0\gamma_\mu)_{\alpha\beta}(\psi^*_\alpha\psi_\beta - \psi_\beta\psi^*_\alpha)\Big]$$
$$= {}^1/_2[(\psi^+\gamma_0\gamma_\mu i\partial\psi/\partial x^\mu + \psi^{C+}\gamma_0\gamma_\mu i\partial\psi^C/\partial x^\mu)$$
$$+ m(\psi^+\gamma_0\gamma_5\psi + \psi^{C+}\gamma_0\gamma_5\psi^C) + eA_\mu(\psi^+\gamma_0\gamma_\mu\psi - \psi^{C+}\gamma_0\gamma_\mu\psi^C]$$
$$= {}^1/_8\sum_\pm\Big\{[(1\pm i\gamma_5)\psi]^+\gamma_0\gamma_\mu i\frac{\partial}{\partial x^\mu}[(1\pm i\gamma_5)\psi] + [(1\pm i\gamma_5)\psi]^{C+}$$
$$\times\gamma_0\gamma_\mu i\frac{\partial}{\partial x^\mu}[(1\pm i\gamma_5)\psi]^C + eA_\mu([(1\pm i\gamma_5)\psi]^+\gamma_0\gamma_\mu[(1\pm i\gamma_5)\psi]$$
$$-[(1\pm i\gamma_5)\psi]^C\gamma_0\gamma_\mu[(1\pm i\gamma_5)\psi]^C) \pm im([(1\mp i\gamma_5)\psi]^+\gamma_0$$
$$\times[(1\pm i\gamma_5)\psi] - [(1\mp i\gamma_5)\psi]^C\gamma_0[(1\mp i\gamma_5)\psi]^C)\Big\}. \quad (15)$$

The Lagrangian (15) is not invariant with respect to P, but is invariant with respect to the combined transformation PC (if C for two-component spinors is defined, for example, by $[(1+i\gamma_5)\psi]\to[(1+i\gamma_5)\psi]^C$). In the case of the choice of Eq. (15) the Fermi statistics is preserved for the anomalous spinors; the Dirac-type equation for ψ^{12} has the form (10), noninvariant under P, and

*Unlike the others, type II spinors ψ^{22}, etc., can be carried over into a five-dimensional space.[2]

that for ψ^{21} has the inversion-noninvariant form (9).

Second, we could renounce the invariance with respect to T′, replacing it with invariance with respect to $T = T' \times C$, and preserving invariance with respect to P. In this case, for example for ψ^{12}, the Lagrangian can be written in one of two possible forms

$$L = {}^1\!/_2 \Big[(\gamma_0\gamma_\mu)_{\alpha\beta} \Big(\psi^*_\alpha i \frac{\partial \psi_\beta}{\partial x^\mu} + i \frac{\partial \psi_\beta}{\partial x^\mu} \psi^*_x \Big) + m (\gamma_0)_{\alpha\beta} (\psi^*_\alpha \psi_\beta + \psi_\beta \psi^*_\alpha)$$
$$+ eA_\mu (\gamma_0\gamma_\mu)_{\alpha\beta} (\psi^*_\alpha \psi_\beta + \psi_\beta \psi^*_\alpha) \Big]$$
$$= {}^1\!/_2 \Big[\Big(\psi^+ \gamma_0 \gamma_\mu i \frac{\partial \psi}{\partial x^\mu} - \psi^{C+} \gamma_0 \gamma_\mu i \frac{\partial \psi^C}{\partial x^\mu} \Big) + m (\psi^+ \gamma_0 \psi - \psi^{C+} \gamma_0 \psi^C)$$
$$+ eA_\mu (\psi^+ \gamma_0 \gamma_\mu \psi + \psi^{C+} \gamma_0 \gamma_\mu \psi^C) \Big]. \tag{16}$$

Unlike the previous ones, the Lagrangian (16) is invariant with respect to transposition. Consequently, the particles described by it must obey not the Fermi, but the Bose statistics (Bose spinors). There is, however, also a possible way of writing the Lagrangian that corresponds to Fermi statistics; thus in this case the unique relation between spin and statistics is lost.

Finally, we only mention the third possibility for constructing a Lagrangian for the anomalous spinors; this is based on doubling the number of components. This possibility has been discussed in earlier papers;[2,7] it also can be associated with a violation of the usual unique realtion between spin and statistics.

In the formation of bilinear (multilinear) expressions account should be taken of the possibility of using spinors of all types (I, II, III, IV, A, B, C, D, etc.) and thus getting unusual combinations. For example, Mirianashvili[12] has used combinations of spinors (A, B, C, D) to obtain bosoms of "imaginary parity," which transform under inversions with a factor ±1.

In an analogous way one can obtain anomalous "bosons," or it is better to say particles of integral spin ("integrons" with anticommuting P and T), by using combinations of mixed and normal spinors. Since one cannot construct from such bosons inversion-invariant combinations, one must resort to doubling the number of quantities, adding a scalar to the pseudoscalar, and so on, these being quantities that are interchanged on reflections. If anomalous integrons are formed by means of anomalous boson spinors, they will be fermions.

In conclusion we present as an example a possible preliminary assignment of particles and fields. We shall suppose that: 1) spinors of the class $\psi^{1A1A} = \psi^e$ describe electrons and positrons; 2) self-adjoint spinors $\psi^{1C1C} = \psi_\nu$, corresponding to zero mass, describe neutrinos;* 3) spinors $\psi^{2A2A} = \psi_\mu$ describe μ mesons. It is also possible in all cases to replace class A by B and C by D. Then we have as an invariant with respect to the strong inversion PC the Hermitian effective interaction (different spinors are assumed to anticommute):

$$\varphi_\pi \psi^+_\mu \gamma_0 (1 - i\gamma_5) \psi_\nu - \varphi_\pi \psi^{C+}_\mu \gamma_0 (1 + i\gamma_5) \psi_\nu.$$

At the same time, for the same reasons that bring about the absence of a mass term in the neutrino Lagrangian, the μ-meson decay will necessarily be of vector or pseudovector character:

$$\psi^+_e \gamma_0 \gamma_\mu (1 + \gamma_5) \psi_\mu \cdot \tilde{\psi}_\nu \gamma_0 \gamma_2 \gamma_\mu i \gamma_5 \psi_\nu + \text{Herm. adj.}$$
$$= {}^1\!/_4 [\psi^+_e \gamma_0 \gamma_\mu (1 + \gamma_5) \psi_\mu (\tilde{\psi}_\nu (1 - i\gamma_5) \gamma_0 \gamma_2 \gamma_\mu (1 + i\gamma_5) \psi_\nu$$
$$+ \tilde{\psi}_\nu (1 + i\gamma_5) \gamma_0 \gamma_2 \gamma_\mu (1 - i\gamma_5) \psi_\nu)] + \text{Herm. adj.}$$

or $\psi^+_e \gamma_0 \gamma_\mu (1 - \gamma_5) \psi_\mu \tilde{\psi}_\nu \gamma_0 \gamma_2 \gamma_\mu \psi_\nu$ + Herm. adj.

Thus according to this scheme if decay of a π meson gives a neutrino with a definite circular polarization, the decay of the μ meson gives two neutrinos with the opposite circular polarization.

From this point of view it is natural to characterize baryons by the anomalous representations,[2] and best simply to assign to them anomalous spinors. Then the first alternative, which is physically the most acceptable, gives a separation of a spinor into two two-component semispinors that do not transform into each other under PC and T′, and this can be associated with the existence of isotopic pairs, for example proton and neutron. This gives a possibility of interpreting the isotopic spin group in the framework of ordinary space.

On the other hand, one could characterize the baryons by spinors of the normal classes, using assignments to the classes A, B, C, D to introduce the baryon number.

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*According to the invariant condition (6), where it is convenient to take $\Omega = \pm i$, so that by fusion of two neutrinos one can obtain fields with the properties of photons.

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